

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:

• Scientific or graphical calculator

Wednesday 19 January 2011 Afternoon

Duration: 1 hour 30 minutes

4725

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the printed answer book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

• Do not send this question paper for marking; it should be retained in the centre or destroyed.

- 1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 2 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Find
 - (i) 2A + B, [2]
 - (ii) AC, [2]
 - (iii) CB. [3]
- 2 The complex numbers z and w are given by z = 4 + 3i and w = 6 i. Giving your answers in the form x + iy and showing clearly how you obtain them, find
 - (i) 3z 4w, [2]

(ii)
$$\frac{z^*}{w}$$
. [4]

3 The sequence u_1, u_2, u_3, \ldots is defined by $u_1 = 2$, and $u_{n+1} = 2u_n - 1$ for $n \ge 1$. Prove by induction that $u_n = 2^{n-1} + 1$. [4]

4 Given that
$$\sum_{r=1}^{n} (ar^3 + br) \equiv n(n-1)(n+1)(n+2)$$
, find the values of the constants *a* and *b*. [6]

5 Given that A and B are non-singular square matrices, simplify

$$AB(A^{-1}B)^{-1}$$
. [3]

- 6 (i) Sketch on a single Argand diagram the loci given by
 - (a) |z| = |z 8|, [2]
 - **(b)** $\arg(z+2i) = \frac{1}{4}\pi$. **[3]**
 - (ii) Indicate by shading the region of the Argand diagram for which

$$|z| \le |z-8|$$
 and $0 \le \arg(z+2i) \le \frac{1}{4}\pi$. [3]

- 7 (i) Write down the matrix, A, that represents a shear with *x*-axis invariant in which the image of the point (1, 1) is (4, 1). [2]
 - (ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. [2]
 - (iii) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$.
 - (a) Draw a diagram showing the unit square and its image under the transformation represented by C.
 - (b) Write down the determinant of C and explain briefly how this value relates to the transformation represented by C. [2]

- 8 The quadratic equation $2x^2 x + 3 = 0$ has roots α and β , and the quadratic equation $x^2 px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.
 - (i) Show that $p = \frac{5}{6}$. [4]
 - (ii) Find the value of q.

9 The matrix **M** is given by
$$\mathbf{M} = \begin{pmatrix} a & -a & 1 \\ 3 & a & 1 \\ 4 & 2 & 1 \end{pmatrix}$$
.

- (i) Find, in terms of *a*, the determinant of **M**.
- (ii) Hence find the values of a for which \mathbf{M}^{-1} does not exist.
- (iii) Determine whether the simultaneous equations

$$6x - 6y + z = 3k,$$

$$3x + 6y + z = 0,$$

$$4x + 2y + z = k,$$

where k is a non-zero constant, have a unique solution, no solution or an infinite number of solutions, justifying your answer. [3]

- 10 (i) Show that $\frac{1}{r} \frac{2}{r+1} + \frac{1}{r+2} \equiv \frac{2}{r(r+1)(r+2)}$. [2]
 - (ii) Hence find an expression, in terms of *n*, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$
 [6]

(iii) Show that
$$\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$$
. [3]

[5]

[3]

[3]

There are no questions printed on this page.

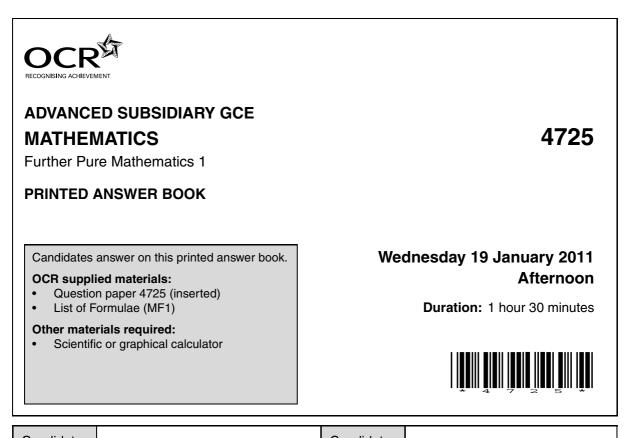


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1 (i)	
1 (ii)	
1 (iii)	

2 (i)	
2 (ii)	

3	3	

	5	
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7 (i)		
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7(iii)(a)		
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8 (i)	
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9 (i)	
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9 (iii)	
10 (i)	

10 (ii)	

11

10 (iii)	



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